ZDM (2018) 50:173–185
https://doi.org/10.1007/s11858-018-0930-8

ORIGINAL ARTICLE

Developing a mathematical modelling course in a virtual learning environment

Daniel Clark Orey[1] · **Milton Rosa**[1]

Accepted: 27 March 2018 / Published online: 6 April 2018

Abstract

This study was conducted during the first semester of 2016, from March 21st to June 30th, with 104 students, in eight educational centers or *polos*, in the states of Minas Gerais and São Paulo. Previously, these mathematics teacher education students had no opportunity to study in higher education in Brazil. They were enrolled in a *Mathematical Modelling* course, as part of the *Universidade Aberta do Brasil* (*UAB*), which is the Brazilian open university, at the *Universidade Federal de Ouro Preto* (*UFOP*). All stages of this study were performed in accordance with case study methodological procedures, which covered data collection and analysis. The interpretation of the results was accomplished through the development of categories, which emerged from collected qualitative data at the completion of the fieldwork. These procedures helped the researchers to answer the research question: *How can technological resources available in a Virtual Learning Environment* (*VLE*) *help students to interact and collaboratively develop mathematical modelling projects that assist them in solving problems they face in daily life?* One important claim for mathematical modelling in a VLE is to favor the development of students in their interaction and collaboration in solving problems they face daily, by elaborating modelling projects through the use of technological resources available in this environment. By developing these projects, students learned to problematize, contextualize, and investigate problems. As well, they prepared questions that aimed to seek, collect, select, organize, and handle the information that allowed them to reflect critically about the role of mathematics in their own context. The results from this study show that the development of modelling projects in a VLE helped students to interact and collaboratively inquire and investigate their chosen theme in accordance with their own interests and reality.

Keywords Case study · Mathematical models · Mathematical modelling · Modelling projects · Virtual learning environments

1 Introduction

The implementation of virtual learning environments (VLEs) enables students, who live in disperse locations, to collaborate, share, and create new opportunities that increase their use of distinct technologies (Clark and Mayer 2011). While enhancing their e-learning experience, the VLE used by the *Universidade Federal de Ouro Preto* (UFOP), supports self-directed and ongoing learning by students who previously had no access or opportunity to study in higher education contexts. For example, they were able to converse and share ideas across large distances, because some of the educational centers are over one thousand kilometers from the campus.

It is important to emphasize that even though this argument may not seem revolutionary to traditional face-to-face educators, in the Brazilian open university context, VLE has become a key element in the democratization of mathematics education and now allows access to opportunities once given only to the elite and small groups of students in fixed locations. Therefore, distance education and VLE have allowed a large portion of the Brazilian population, who were traditionally denied access to quality higher public education, to advance both personally and professionally.

In accordance with this context, Balaji and Chakrabarti (2010) argued that "important payoffs of using technology tools include flexibility, convenience and accessibility for

✉ Milton Rosa
milton@cead.ufop.br; milrosa@hotmail.com

Daniel Clark Orey
oreydeema@gmail.com

[1] Universidade Federal de Ouro Preto, Ouro Preto, Brazil

students to complete their learning anytime and anyplace" (p. 2). For these reasons, Shepherd et al. (2016) argued that the implementation of online teacher education programs is expanding at different educational levels.

At the beginning of the 21th century; Kerr et al. (2002) stated that the association of VLEs with technological resources fosters interactions among professors, students, tutors, and the teaching materials because they offer tools that help educators to schedule a range of learning activities rather than just managing the course content. In this regard, Rosa and Orey (2017) argued that professors can prepare instructional materials and strategies, while tutors provide assistance at both the *educational centers* or *polos*[1] and the university; at the same time technological tools encourage interaction between participants in the VLEs.

This is especially important in the context of the *Universidade Aberta do Brasil* (*UAB*), which is the Brazilian open university, as we offer courses to a population previously excluded from higher education. Consequently, the incorporation of

> (…) internet and digital technologies into education is paramount if we want education to be coherent with what being human has come to mean for those who are regular internet users. For those without internet access, schools could be the first entry door into the virtual world. (Borba 2012, p. 805)

According to this assertion, online education has emerged as one way for students and faculty to collaborate and to use new forms of media. For example, the results of the study conducted by Reese (2015) showed that a balanced online environment provides a blend of both asynchronous (emails and forums) and synchronous (chats and webconferences) learning opportunities, which promote communication and collaboration between students and professors.

Similarly, Dillenbourg (2000) also argued for the importance of VLEs in the development of *social interactions*[2] through synchronous and asynchronous communication, audios and videos, and indirect communication such as sharing objects. Thus, VLEs can be considered as the "mutual action between professors and students [and tutors], in environments whose uniqueness is separation from each other, and as a result often exhibits unique behavior patterns in relation to distance education" (Moore and Kearsley 2005, p. 224).

[1] Educational centers or *polos* are extensions of the university maintained by local governments. They are equipped with computer labs, internet, libraries, and tutorial assistance.

[2] It is important to highlight that there are theories related to distance education such as the *Theory of Transactional Distance* (Moore 1993) that explain how interaction is critical to the student success in the VLE.

Social interactions are important variables that decrease transactional distance between professors, tutors, and students; as well as increasing learning effectiveness (Moore 1993). Effective distance teaching and learning depends upon the nature of social interactions and how they are facilitated through the use of technological resources (Moore and Kearsley 2005). In this context, VLEs become democratic educational environments that enable the development of collaborative learning and dialogic interactions between professors, tutors, and students (Rosa and Orey 2016).

The main components of VLEs used at UFOP include curriculum mapping (breaking curriculum into sections that can be both assigned and assessed), student tracking, online support to professors, tutors, and students, electronic communication (e-mails, forums, webconferences, and web publishing), and internet links to outside curricular resources. It is also necessary to emphasize that VLEs are not limited to spaces for the replication of rigid and expository pedagogical practices where knowledge is transferred between students regardless of the use of the media that intervene in this process (Borba and Villarreal 2005).

VLEs provide technological resources that enable professors, tutors, and students to participate in online learning interactions regardless of time or distance. These resources such as forums are supportive educational tools that enable modifications in the mathematics curriculum, teaching methods, personalized assessment, medium of communication, and educational resources in order to cater to particular students' needs and differences in learning (Silva et al. 2007).

Therefore, when "managed effectively, discussion forums encourage students to share information, build on the ideas of others, and construct understanding" (Silverstone and Keeler 2013, p. 18). According to Cottrell and Donaldson (2013) accessibility of technological resources increases the interactions between students and content. This is especially true in the UFOP context where many students travel up to an hour to study in their educational centers because they have limited access to internet in their homes.

In a previous study, Borba et al. (2010) argued that the use of technologies such as computers, software, and internet change the way knowledge is produced through collaboration. Thus, the internet can be considered a medium that transforms the mathematical practices of both professors and students involved in online mathematics education courses.

In online courses, many professors use a combination of videos, figures, written texts, and sounds. These diverse ways of combining different tools is informed by the notion that knowing can be understood as the development of multiple representations. This diversity of representations encourages coordination between visual, graphic, tabular, and symbolic representations, and verbal communication, which is needed to explore problems in order to determine ways for teachers to examine a multiplicity of solutions

(Borba 2012). This means that "students using digital tools are able to employ multiple and different methods to solve a problem" (Greefrath and Rieβ 2013, p. 453).

The results of the study conducted by Silverman (2012) demonstrate the importance of the development of mathematical knowledge through teaching in online environments that focus on students' engagement in collaborative activities. In a previous study, Silverman (2011) found that online collaboration that includes group discussions and emergent conversations focusing on issues related to teaching and learning mathematics, holds the potential for supporting the development of mathematical knowledge.

Since technology *opens windows into mathematics* and because multiple solutions to a problem can be investigated based on the coordination of multiple representations and modes of communication (Borba 2012), VLEs incorporate online mathematical practices through the use of pedagogical approaches that integrate experimentation, interaction, and the formulation of conjectures, which allow for the development of rich discussions in the modelling process.

Borba (2012) further argued that "technological tools may also be seen as catalysts, providing opportunities and tools for collaboratively exploring more complex mathematical ideas" (p. 804), which are central to the elaboration of modelling projects in VLEs. This was especially important to our context with students living in a diversity of locations throughout the country. Because of this, we feel confident that "online mathematics teacher education (…) is characterized as an emergent area of research in mathematics education" (Borba and Llinares 2012, p. 697).

In this context, the main objective of this study was to answer the following research question: *How can technological resources available in a VLE help students to interact and collaboratively develop mathematical modelling projects that assist them in solving problems they face in daily life?*

According to this research question, the responsibility of the researchers was to create a VLE that offered a series of modelling activities supported by interactive and collaborative features, along with the technological resources available in this environment, that allowed a diversity of students to communicate successfully with their professors, tutors, and peers, in order to develop their modelling projects.

2 Mathematical modelling and VLEs

Mathematical modelling offers several perspectives that have been widely discussed and debated in congresses, conferences, seminars, and in national and international events. One of these perspectives deals with the development of mathematical modelling in VLE contexts (Rosa and Orey 2016), which forms an emerging issue in mathematics education research.

These technological resources not only broaden "the range of approaches that can be taken to solve certain mathematical models, but also the types of situations that can be investigated by providing the possibility of using solution strategies" (Greefrath and Rieβ 2013, pp. 445–446), but also help students to discuss, reflect, and make predictions about the phenomenon under study through the elaboration of models that represent these situations (Rosa and Orey 2017).

Therefore, online environments are important for the ongoing growth and improvement of mathematical modelling processes in regard to student interactions that promote both collaboration and participation, and which enable them to reflect, to engage in dialogue, to make decisions, and to assume critical positions in relation to the modeled phenomenon (Parra-Zapata et al. 2017). In this context, VLEs help to "improve students' learning outcomes by contributing to their intellectual growth and critical thinking" (Balaji and Chakrabarti 2010, p. 2).

With respect to interactions created by VLEs, Parra-Zapata et al. (2017) designed an online teaching training environment to work specifically with mathematical modelling for prospective teachers. Their proposal aimed to promote students' interaction in different working tasks in both synchronous and asynchronous communication. The results of their study showed that this environment facilitated participation, interactions, and collaboration during the development of the modelling process.

The general design of the course developed by Parra-Zapata et al. (2017) included eight synchronous sessions of four hours each for the development of modelling themes. In these sessions, the *WizIQ* and *Moodle* platforms were available to the students. This design promoted the participation of students in different technological resources of a VLE. In addition, a set of asynchronous activities were elaborated, prior to the synchronic intervention, such as analysis of modelling themes, construction of wikis, and the solution of modelling tasks.

Technological resources available on the VLE provide opportunities for students to interact collaboratively in their learning process through the development of modelling activities associated with the elaboration of models. This approach helps students to develop their autonomy when they are searching, selecting, organizing, and reflecting in relation to their chosen themes (Rosa and Orey 2016).

Similarly, Haris (2017) developed a VLE that encouraged students to study collaboratively in order to build their own mathematical modelling practices in schools. The main objective was to assist and support the improvement of mathematical modelling abilities and the development of autonomy in the students through the use of technologies.

The results of the study conducted by Pereira (2015) highlighted that this approach is related to the development of students' autonomy as they learn how to elaborate their own mathematical strategies necessary to the advancement of the modelling process. Previously, the results of the study conducted by Rosa and Orey (2016) showed that teachers can develop student autonomy by elaborating mathematical activities or modelling projects based on the procedures and practices they experience in their daily lives.

Previously, Malheiros (2008) conducted a study in regard to the elaboration of modelling projects by mathematics teachers in a distance extension course through VLE. The results showed that collaborative work was a factor that facilitated the elaboration of the modelling projects developed through dialogue and by using technological media available to participants.

In another study, Pereira (2015) investigated an extension course for mathematics teachers in relation to mathematical modelling processes with the specific objective of studying possibilities of its development in distance learning. The results showed that distance education favored the improvement of teaching knowledge that promoted the development of autonomy in regard to the execution of the proposed modelling activities.

In this context, learning mathematics is related to the process of construction of mathematical knowledge with the development of the modelling process in online environments. This approach uses the interests and motivations of students to learn (Rosa and Orey 2016) by applying digital tools in many phases of the modelling process (Greefrath and Rieβ 2013). These features have enabled the development of a variety of educational and technological methodologies that provide needed support in the achievement of mathematical modelling activities in VLEs (Rosa and Orey 2017).

Therefore, the main elements of mathematical modelling activities included in the instructional process that occur in the VLEs are related to developing innovative ways of applying mathematics by addressing daily phenomena, searching for information to collect real data, discussion by peers to propose the elaboration of mathematical formulas in order to critically and reflexively analyze the models, and use of empirical data to determine solutions to the problems faced by society.

3 Methodological design and data collection

Among various qualitative methods available for research in mathematics education, the researchers have chosen the *case study* design. The rationale for using a case study design in this study is that the

> (…) real business of a case study is particularization, not generalization. We take a particular case and come to know it well, not primarily as to how it is different from others but what it is, what it does. There is emphasis on uniqueness, and that implies knowledge of others that the case is different from, but the first emphasis is on understanding the case itself. (Stake 1995, p. 8)

According to Yin (1994), case study design is a qualitative research approach that investigates a contextualized and contemporary phenomenon within a specified boundary. Thus, case study is an exploration of a *bounded system* (a case) over time through detailed in-depth data collection involving different sources of information rich in context (Creswell 2002).

This instrumental case study using a paradigm developed by Stake (1995) was *bounded* by time and space because it was conducted over the first semester of 2016 in the eight educational centers in two states in Brazil. The main focus was to develop a broader understanding of the phenomenon related to how available VLE technological resources encourage students to interact and collaboratively develop modelling projects relevant to solving problems they face in their own context.

The characteristics of this case study included examining a particular phenomenon, bounded in time and space, which the researchers chose, by providing a detailed description of contextual information in this setting, and gathering extensive material from multiple sources to provide an in-depth picture of the case (Creswell 1998). The aim of this methodological design was to help the researchers examine collected data in order to gain insights and an understanding of the research question.

3.1 Data collection

Data were collected during the first semester of 2016, from March 21st to June 30rd, with 104 prospective mathematics teachers from eight long distance educational centers of *UFOP*,[3] in the towns of Araguari, Barão de Cocais, Caratinga, Conselheiro Lafaiete, João Monlevade, Lagamar, and Passos in the state of Minas Gerais, and in the town of São José dos Campos in the state of São Paulo, in Brazil. The authors of this article were the only researchers/professors involved in the development of this course, and who teach mathematical modelling via distance education at UFOP.

[3] Besides traditional face-to-face degrees, the university offers, via long-distance, teacher education degrees in geography, mathematics, and pedagogy as well as a Bachelor's degree in public administration. The long-distance teacher education program in mathematics has been offered since the second semester of 2007.

Various sources of data were collected that enabled the description in sufficient detail to provide an accurate account of field experiences, and their placement in a context with multiple dimensions and realities in a holistic view (Holloway 1997). Hence, data collection was performed in relation to the fieldwork using multiple sources of data including webconferences, discussion forums, mathematical modelling projects, modelling activities proposed in the VLE, and researcher fieldnotes. Borba (2012) argued that data collection in online environments occurs using the retrieval devices that automatically record videos, texts, and sounds.

3.2 Data analysis and interpretation

After data collection was completed, the researchers began analysis that consisted of the examination, codification, and categorization of evidences (Yin 1994), guided by the initial propositions of the study.

The qualitative data analysis was performed using information collected during observations of student interactions in the development of the proposed modelling activities. Analysis was also carried out by observing the elaboration of the models and modelling projects developed, using technological tools available in this VLE.

After data collection, the researchers began data analysis that consisted of the examination, codification, and categorization of the evidence that guided initial propositions of the study (Yin 1994). With the completion of this systematization, the researchers reflected on the information contained in the data in order to interpret the results obtained in this analytical process. This approach enabled us to enunciate a response to the research question of this study.

For the accomplishment of the analytical and interpretative processes, the codification of the qualitative data was developed through the elaboration of categories. The main objective of data reduction was to reveal the most important features of the analyzed information by categorizing them into groups that had the same theoretical conceptualizations.

3.3 Methodological procedures

The observations developed during the accomplishment of modelling activities, discussion forums, and webconferences along with the elaboration of the modelling projects were recorded by the researchers in their fieldnotes, which comprised information obtained in the data collection process. Webconferences were also recorded and were available to all participants in the course. This approach enabled the researchers to use these technological tools to revisit information provided about the student modelling process.

During the accomplishment of the modelling activities, the researchers also recorded their own notes related to their observation of the behavior of the students participating in the VLE. They also observed the level of interaction and collaboration related to the discussions that occurred in the forums and webconferences, as evidenced by questions and comments posted on the VLE, which contained important information that assisted students in analyzing and interpreting the findings of their mathematical models.

The results reported in this investigation come from one case study composed of *Groups A, B*, and *C*, with four students each, in the São José dos Campos educational center. The researchers justify this option because saturation occurred during the analytical process of a larger study conducted with 104 students in eight educational centers at the university. It is important to emphasize that Guest et al. (2006) argue that saturation often occurs around 12 participants in homogeneous groups.

Thus, the researchers found that no new descriptive codes or thematic categories were emerging from the data analysis; also, no additional new information was attained (O'Reilly and Parker 2012) in other educational centers.

4 Designing a VLE for mathematical modelling

The organizational features of the course *EAD512—Seminar I: Mathematical Modelling* were designed to facilitate and encourage interactive and collaborative work through the use of the technological resources available on the VLE. These resources enabled students to organize their learning experience in such a way that mathematical knowledge could be used and placed in real-world contexts they understood.

The course syllabus, posted on the VLE, contained the description of the course, the terms of the proposed tasks in relation to the modelling activities, and the calendar with dates and times of synchronous and asynchronous activities. The curricular program of this course is comprised of both theoretical and practical mathematical content that enabled the development of the modelling process in this environment.

The syllabus also described the epistemological characteristics and conceptions of modelling, the connection between modelling and ethnomathematics, the critical and reflective dimensions of modelling, the elaboration of models, the development of the mathematical modelling process in the classrooms, and the role of mathematical modelling in teacher education programs.

The course as developed by the researchers included three activity modules that contained the relation between three phases and ten stages of the development of the modelling process proposed by Rosa (2000). Table 1 shows the modules of activities posted on the VLE and its relation to the three phases and the ten stages of the development of mathematical modelling projects.

Table 1 Activity modules for mathematical modelling on the VLE

Phases	Period	Stages
Module 1: Initial phase modelling preparation	From March 21th, 2016 to April 16, 2016	1 Choosing a theme 2. Researching about the theme
Module 2: Intermediate phase Modelling project development	From April 23rd, 2016 to June 18, 2016	Subphase 1: formulation of mathematical problems 3. Elaboration of the questions 4. Formulation of mathematical problems Subphase 2: mathematical models 5. Elaboration of mathematical models 6. Resolution of mathematical models. Subphase 3: interpretation and comparison 7. Interpretation of the solutions 8. Comparison of the model with reality
Module 3: final phase presentation of the report	From June 20th, 2016 to June 30th, 2016	9. Modelling project presentation 10. Evaluation

Table 2 Modelling activities proposed in modules 1, 2, and, 3

Modules	Modelling activities
Module 1 Initial phase Modelling preparation	Video/web conference to explain to the students the dynamics of the modelling process Organizing working groups Choosing a theme Researching the theme Video lesson about the elaboration of the modelling projects Submission of the research report that describes the development of the investigation Discussion forum on the chosen theme Discussion forum on the justification of the chosen of theme Submission of the revised justification based on the comments made by the researchers and tutors Submission of the reflection text based on the proposed activities of module 1
Module 2 Intermediate phase Modelling project development	Submission of the report on the formulation of three questions in regards to the chosen theme Video/webconferences with professionals from various sectors about the chosen themes such as water pollution, air pollution, and water consumption reduction Video/web conference to discuss the elaboration of mathematical models Video lesson about the elaboration of mathematical models Submission of the report on the elaboration of mathematical models Submission of the revised mathematical models in accordance to the comments provided by the researchers and tutors Submission of the reflection text on the interpretation of the solution and on the comparison of the models with reality Submission of the reflection text on the activities proposed on module 2
Module 3 Final phase Presentation of the report	Video/web conference to advise students during the development of their projects and guide them to communicate correctly and positioning themselves in presenting, and defending their projects Video lesson about the presentation of the mathematical modelling projects Discussion forum on the elaboration of the final report Presentation and defense of the final report of the mathematical modeling project to a committee in each *educational center* Submission of the final report on the mathematical modeling project Submission of the reflection text on the elaboration of the modelling projects Submission of the reflection text on the mathematical modelling process Submission of the final report of the mathematical modeling project

In these three modules, the modelling activities were related to group reflection on assigned texts and tasks, the process of the elaboration of models, and the development of the modelling projects. Table 2 shows the modelling activities proposed in modules 1, 2, and 3.

It may be useful here to describe briefly the three phases and the ten stages of the development of the mathematical modelling projects.

4.1 Initial phase: modelling preparation

This phase included two stages: (1) *choosing* and (2) *researching a theme*. The researchers met with the students via webconferences. During these meetings the professors were on the main UFOP campus and the students were at their educational centers.

The first meeting was held from 7 pm to 9 pm, on March 21st, 2016. During this time, the researchers explained to the tutors and students the dynamics of the modelling process and then presented the themes that were related to water pollution, air pollution, and water consumption reduction.

From March 22nd, 2016 to April 3rd, 2016, students were organized in working groups and chose their themes. Twenty-six groups of four students each were organized at the eight educational centers in two states. Ten groups chose a water pollution theme, nine groups chose an air pollution theme, and seven groups chose a reduction in water consumption theme. At the next webconferences, the researchers demonstrated to the students how to collect quantitative and qualitative data effectively by searching for information on their chosen themes.

Both researchers and *distance tutors* at the university discussed how the students needed to justify their themes. *Face-to-face tutors*, located at each educational center, helped students to organize study groups and to develop their models and projects further. Students also watched a *video lesson*[4] about the elaboration of the modelling projects.

4.2 Intermediate phase: mathematical modelling development

In this phase, student groups informed researchers and tutors about their progress in relation to the type of research they were developing in regard to their chosen themes. They also indicated bibliographic references and data sources that they used to conduct their research by using specific links on the VLE; they also recorded the main ideas related to their themes, when they were reading required materials.

Researchers and tutors acted as facilitators of the modelling process by organizing lectures via webconferences with professionals from various sectors. The content of these lectures was related to the ongoing process and themes chosen by the students such as water pollution, air pollution, and reduction in water consumption.

It is important to emphasize that this phase consisted of three subphases and five stages (Rosa 2000).

4.2.1 Subphase 1: formulation of mathematical problems

This subphase included two stages: (3) *elaboration of the question* and (4) *formulation of mathematical problems*, which helped students understand and comprehend environmental phenomena around them in holistic ways. For example, participants in *Group A* posted, on the VLE, the following question: "How can water reuse be improved?".

To develop their mathematical model, which dealt with the percentage of water used in relation to the total cost of its production, the students then formulated the following mathematical problem:

> If the average productivity of water, in terms of the value of production used per cubic meter, is higher for organizations that reuse water (R\$[5] 6657/m^3) in comparision to others that do not (R\$ 4103/m^3), then the following question arises: is the production value for organizations that reuse water higher than those that do not adopt this practice?

Thereafter, student groups, in the forums, talked about the elaboration of the questions and the formulation of mathematical problems that they developed, and for which they received feedback from the researchers and tutors. For example, Balaji and Chakrabarti (2010) affirmed that online discussion forums provide relevant opportunities for students to actively engage in their learning process through active participation.

4.2.2 Subphase 2: mathematical models

This subphase included two stages: (5) *elaboration of a mathematical model* and (6) *using mathematical models to solve real problems* by using the technological resources available in the VLE. For example, Greefrath and Rieβ (2013) argued that the solution of modelling problems often makes use of digital tools that require that problems students devise be understood and then translated into mathematical language.

[4] A *video lesson* is a recorded lecture posted online and distributed in a video form. In this study, lectures about the development of the elaboration of the modelling projects were recorded using video, audio or both and then uploaded and made viewable on the VLE for the convenience of the students. It is necessary to highlight that video lessons are widely used in distance education with the aim of illustrating, reinforcing, and complementing the content of the course. Video lessons are important didactic resources that assist professors in the elaboration of content.

[5] R\$ is the symbol of the Brazilian currency called the real.

Groups posted reflective texts explaining the progress and difficulties they encountered during the development of their modelling projects. In the forums the students discussed the mathematical content needed for the development of their models. Students also participated in webconferences in their educational centers in order to clarify questions and difficulties in regard to the elaboration of their mathematical models and projects.

In this context, Rosa and Orey (2017) argued that with discussion forums and webconferences, professors, tutors, and students are empowered to analyze critically the interactions enabled by these tools, which contributed to the critical-reflective development of the elaboration of mathematical models in this VLE.

4.2.3 Subphase 3: interpretation and comparison

This subphase included two stages: (7) *interpretation of the solutions* and (8) *comparison of models with real contexts*. In the forums, groups discussed the interpretation of the solutions and the comparison of their models with reality. Subsequently, they posted reflective texts on the development of these two stages. Students also began posting revised models after researchers, tutors, and peers shared recommendations. Then, in the forums they discussed the revisions addressed in their mathematical models.

4.3 Final phase: presentation of the report

This phase included two stages: (9) *modelling project presentations* and (10) *evaluation*. It was completed on June 30, 2016 with the presentations of the projects by the groups of students via webconferences. Previously, they posted their final reports via word documents and their *PowerPoint* presentations on specific links in the VLE. Before the presentations, students discussed in the forums their concerns and queries related to the preparation, presentation, and defense of their final reports.

By conducting webconferences, researchers advised students during the development of their projects and guided them on how to communicate correctly and position themselves in presenting and defending their projects. After the presentations, students' projects were evaluated by members of a committee who participated in the webconferences together with face-to-face tutors at each educational center where the defenses took place.

5 Discussing results

The analytical process developed in this study helped the researchers to elaborate four categories: *Distance Education and VLE, Critical and Reflective Dimensions of Mathematical Modelling, Environmental Education*, and *Mathematical Modelling Projects in the VLE.*

5.1 Mathematical modelling projects on the VLE

This category was chosen to be discussed here because of its relevance in answering the research question of this study. For example, the results in this category showed that the combination of mathematical modelling projects and the technological resources available in the VLE enabled students to research their themes in an interactive and collaborative manner by sharing inquiries and exchanging information with researchers, tutors, and peers in the discussion forums and webconferences.

In this context, students in *Group A* stated that the "VLE helped us to share information because mathematical models were debated and discussed through the exchanging of suggestions that occurred in the forums". Similarly, students in *Group C* affirmed that the "use of discussion forums facilitated the learning of mathematical content and the development of collaboration during the modelling process". In a webconference, students from *Group B* argued that "modelling work in the VLE served to assist our group in determining new ways to apply mathematics to our daily lives in a critical and reflective way".

In this particular VLE context, the students learned to investigate problems in their own communities by using a variety of technological resources, in order to enable them to elaborate mathematical models of problems they face in their own communities. For example, during the development of their modelling project, students from *Group C* argued that the "elaboration of mathematical models is a way to detect water pollution in our environment and we need to be aware of this serious problem society is facing so that future generations will not suffer from pollution".

In regard to the mathematical modelling process, students from *Group A* in their participation in the webconferences commented as follows:

> (...) first, we discussed in the forums the identification of a theme in regard to a problem we are facing in our own community. So, we applied a questionnaire to research the opinion of the people from the community in order to identify the problem and, then, we started to research it to obtain information to develop our mathematical models. Technological tools available in the VLE helped us to research our theme related to water consumption reduction to elaborate our mathematical models.

The excerpt from the modelling project developed by students from *Group A* shows that they became aware of the relevance in relation to the connection between mathematics and their daily experiences. For example, the students commented that the:

> (...) development of modelling in the VLE has changed my vision of how mathematics can become more motivating and interesting in learning mathematical content when we relate it to our daily experiences by using technological tools to achieve this goal. This approach guarantees meaningful learning experiences because it is associated with contextualized information acquired in other contexts through interaction and collaboration.

This comment shows how technological resources available in the VLE facilitated the investigation of problems taken from the students' own reality through the elaboration of contextualized modelling activities that helped them to develop their mathematical knowledge through interaction and collaboration. For example, students from *Group A* shared in the forum that "during the development of the modelling process in the VLE, collaboration and interactivity were developed in regard to conversation with our colleagues in the forums, which helped us to make connections to environmental phenomena that occur in our daily life".

An excerpt from the modelling final report elaborated by students from *Group B* showed that "modelling is a teaching and learning strategy used as a way to break existing paradigms between school and real-life mathematics" while students from *Group A* shared in the forum that the "modelling process is related to the development of investigations about themes that come from real life".

These examples taken from students' daily lives were important components of the modelling activities posted on the VLE web site, because they were related to environmental issues faced by society. For example in a webconference, students from *Group B* stated that "we work with societal problems and real data, thus, we can interpret them to help us to reflect on how our attitudes can be changed so that we can develop ourselves as citizens concerned about environmental issues faced by society".

Similarly, students from *Group C* shared conjectures in a discussion forum by stating that the "results obtained from our mathematical model may help people to be aware of the necessity to develop preventive measures to mitigate the damages caused by water pollution in order to protect the environment". In this context, students from *Group A* argued that "modelling can be considered as a system where we can describe a real problem mathematically in order to predict future results by using technological tools available in the VLE".

In this regard, we provide a concrete example of the elaboration of a model developed by students from *Group A* using technological resources available in the VLE. In their modelling project, the students developed the theme of *Water Reuse* in relation to one of the automotive companies located along the *Paraíba do Sul River*, in the state of São Paulo, Brazil. Students worked collaboratively in the forums to develop their model, as well as in the conclusion of their final report. Thus, they stated that "due to the increasing demand for water in industrial use, several Brazilian regions already face problems related to scarcity and degradation of water resources".

The students also justified, using their own data and models in their final report, that problems in relation to the increasing demand for water in regard to industrial use are particularly critical in large urban centers where rapid demographic and industrial growth have not been accompanied by measures to control land occupation and by implementing an adequate sanitation infrastructure. They also commented that this "process does not help the preservation of natural resources because it does not reduce consumption and water waste".

By using the internet, the students collected data about this automotive company. The information they gathered in their research showed that this company "achieved a 20% reduction in water consumption through water reuse in 2015, which generated about 330 thousand liters of reduction in water consumption". Students also argued in their final report that the "reuse of water is a viable alternative for its rational use and for reaching the goals proposed for its savings". Then, by using the forums, students both discussed and explained their proposed mathematical model:

> (...) by analyzing the number of liters of water that this automotive company in São José dos Campos was able to reduce in 2015 with the water reuse, a model was proposed for the calculation of the amount of money, in reais[6], that this company saved by using this approach. The development of this mathematical model was defined by the application of the following formula in order to identify the payment related to the amount of water captured from the river: $TV = FR \times CC \times UPP$. In this formula, TV is the total value charged by the use of the water; FR is the flow rate; CC is the catchment coefficient that is defined by the sanitation company, which developed a sanitation coefficient table for this purpose; and PPU, which is the unitary public price.

In their research about this theme, students learned that, for this company, the sanitation coefficient of 0.4; the *UPP* of 0.02, and the catchment coefficient of 0.008 were established by the state government. Therefore, the automotive company had a 20% reduction in water consumption in the year of 2015, which comes to 330 thousand liters of water. In their mathematical modelling project posted on

[6] Reais is the plural form for Real, which is the Brazilian currency.

the VLE, students of this group also described the steps they used to perform the calculation used in their mathematical model:

> The first step was to calculate the total amount of water captured and the amount of water spent without its reuse. So, to calculate this value, we had to determine 20% of the 330 thousand, which was the amount of money the company had saved. Then, we performed $P \times 100 = C \times I$ in which P is the amount of 330 thousand liters of water, C is capital that has to be found by applying the rate of 20% and the value of 100, which is constant. Then, these values were placed in the giving formula: $330 \times 100 = C \times 20$. By calculating the capital, we arrived at the amount of water that was used without the reuse, which is 1 million and 650 thousand liters.

The excerpt from the final report of their modelling project showed that the

> (…) catchment coefficient was 0.008 m^3 and the amount of water was 1 million and 650 thousand liters and by multiplying them we obtained the value of R$ 13,200.00. The second step was to calculate the total amount of water charge without its reuse. By applying the TV formula, we determined the value of R$13,200.00 times the catchment coefficient multiplied by the UPP, which provided the amount of R$105,600.00, which was calculated without water reuse. The third step was to calculate the total value of the water charge for its use with water reuse. So, they found that 1 million and 650 thousand minus 330 thousands of water reused is equal to 1 million and 320 thousand liters of water. Thus, the value of the catchment coefficient was 105,600 and by placing it into the TV formula, we came out with 84,480. Then, we determined both the value without and with the water reuse. As it was proposed to find the amount of money that the company had saved in cash, we calculated that the total amount saved was R$21,120.00.

In analyzing this result in the forum, students in this group discussed that the:

> (…) results of this analysis showed that the practice of reuse in this company is associated with large water consumers. In fact, investments in water reuse technologies can lead to significant reductions in costs that are associated with water catchment and its treatment and/or its disposal of effluents. Thus, in this process, the company was able to return used water to the Paraíba do Sul River, with approximately 98% purity by applying the appropriate treatment. Therefore, this model supports the claim that water reuse is beneficial because with the use of cleaner technologies, the company and society benefit as a whole.

At the end of their modelling project presentations via webconference, students argued that “water reuse, besides being a relevant economic factor for companies, benefits communities and society in general”. They also stated that it is “important to prompt users and/or polluters to rationalize the use of this natural resource by maintaining a balance between its availability and demands in order to provide environmental conservation, which is conducive to the well-being of society”.

It is also necessary to state that many students found it hard to adapt to a less autocratic and more open and democratic form of communication between all participants in the VLE because they had to manage time effectively in order to participate in the discussion forums, webconferences, and posting modelling activities. For example, the insertion of modelling projects in the VLE presented a challenge for students because it created new knowledge parameters stemming from the use of new practices through technological resources. However, through the use of shared technological resources available in the VLE, and as students developed confidence, the elaboration and diversity of their mathematical models improved markedly. They became accustomed to a deeper critical reflection on the environmental problems they face in their own communities.

This VLE context also helped the researchers to move from a traditional *chalk and talk* approach to teaching that involves a much more *active* approach to learning. For example, while the students worked in small groups in their educational centers, they also shared questions and solutions in relation to their mathematical models, in the forums and webconferences with other colleagues, groups, and tutors in different distant locations. These technological resources facilitated the investigation of problems taken from the students’ own reality and context that helped them to develop their mathematical knowledge through interaction and collaboration.

An important theme that emerged from this study is the importance of communication. For example, professors and tutors needed to overcome difficulties such as internet connections that impacted the way they communicated with students about the elaboration of their modelling activities. Thus, there was a need to use diverse technological tools for communicating with the students, such as webconference, emails, facebook, and planned visits to the educational centers in order to deliver the modelling content while making it engaging for the students.

The VLE allowed both researchers and tutors to provide positive, meaningful, and constructive feedback through asynchronous and synchronous discussions that reduced much of the anxiety and many of the concerns of students in order to enhance their work. It also played an important role in the development of interactions, collaborations, and creative solutions to their models. These strategies helped the development of communication between researchers, tutors, and students, which improved the process of elaboration of models, with which initially many students found difficulty in coping.

There are some specific aspects of the VLE that were considered to enable students to interact with their peers, tutors, and reseachers in order to foster mathematical modelling activities such as collaborative work, interaction, and access to information and data. These are as follows:

- Assignments that enabled researchers and tutors to check, grade, and provide feedback on uploaded files (text documents, spreadsheets, images or audio and video clips) in the VLE about the development of the models and modelling projects.
- Discussion forums and webconferences that allowed students to participate in asynchronous and synchronous discussions to complement their assignments.
- Video Lessons for delivering the models and modelling in flexible ways.
- Files posted in specific links in the VLE provided by the researchers as course resources related to the models and modelling projects.

This particular form of pedagogical action on the VLE web site directed students towards the development of sound arguments based on data that they collected and towards observing, generalizing, and applying mathematical models to solve future problems faced by their own communities. Also, in a context where information is often unreliable, it helps individuals to move away from emotional manipulation (via social media and *fake news*) and to learn to search for deeper explanations and distinct ways of reflecting and dealing with societal issues based on the scientific method and factual data.

6 Final considerations

The organizational features of the course *EAD512* encouraged interaction and collaborative work through the use of technological resources that enabled our students, who are often of the first generation to attend university and who live in a diversity of locations far from traditional university campuses, to elaborate and work collaboratively on modelling projects.

The VLE used by UAB/UFOP has benefits such as the posting of activities and materials in this environment, participation in the forums, collaboration, and a certain time–space related flexibility in accomplishing tasks, which enables students to engage in threaded discussions that support their progression in mathematical modelling. It provides an affordable environment for students living in a diversity of locations, to share and develop their ideas and models, in an atmosphere in which they feel safe and successful in communicating their growing understanding of mathematical ideas with the participants in this environment.

This VLE was configured as an environment that facilitated the exchange of information and knowledge and facilitated the sharing of questions and a search for alternative strategies to work with mathematical models they themselves developed. For example, in the forums, students debated questions in relation to the elaboration of their models and modelling projects. Thus, technological resources available in the VLE enabled students to collaborate and to develop autonomy and freedom in the exploration of their models.

In this environment, the cooperation between researchers, tutors, and students established relations of mutual respect and support that favored the process of elaboration of their modelling projects. Thus, VLE has altered the relations among them and improved the dynamics of their interactions through technological resources, thus enabling students to share their discoveries and experiences with other participants across distance and time.

This approach favored the development of deeper student reflection on the role of mathematical models in society. It allowed them to discuss new applications in their decision-making process related to environmental problems they confronted in their everyday life. It also provided students with mathematical tools based on the scientific method that allowed them to look at the data and be less manipulated by *fake news*, emotion or propaganda in relation to their chosen themes.

Thus, students learned to focus on the data, and were empowered to look deeply at the data instead of the emotion around their themes. They also learned to develop dialogue towards the solution of local problems based on data they collected. This environment enabled the delivering of modelling activities, with the aim of involving students in the elaboration of projects, which engaged them in analyzing environmental problems through the development of their models.

The configuration of this course allowed the integration of multiple media, presenting information in an organized way, and developing socialization among researchers, tutors, and students. Thus, technological resources helped students to interact collaboratively with colleagues they would never have met and to develop models that assisted them in solving problems they face in their own communities.

In this context, technological resources provided and encouraged the development of research and the acquisition of necessary information for the analysis and interpretation of data collected during the conduction of their investigations. During this process, students demonstrated that they had learned the importance of organization in the accomplishment of their modelling activities, which developed as they shared ideas, strategies, and techniques.

Finally, what makes this experience particularly valuable is that our students, who traditionally have had limited or no access to higher education, were given a rich opportunity, encouragement and the mathematical tools to develop sophisticated models to resolve problems that are relevant to the members of their own communities.

References

Balaji, M. S., & Chakrabarti, D. (2010). Student interactions in online discussion forum: Empirical research from 'media richness theory' perspective. *Journal of Interactive Online Learning, 9*(1), 1–22.

Borba, M. C. (2012). Humans-with-media and continuing education for mathematics teachers in online environments. *ZDM Mathematics Education, 44*(6), 801–814.

Borba, M. C., & Llinares, S. (2012). Online mathematics teacher education: Overview of an emergent field of research. *ZDM Mathematics Education, 44*(6), 697–704.

Borba, M. C., Malheiros, A. P. S., & Zulatto, R. B. A. (2010). *Online distance education*. Rotterdam: Sense Publishers.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization and experimentation*. New York, NY: Springer Science + Business Media.

Clark, R., & Mayer, R. E. (2011). *E-learning and the science of instruction [electronic resource]: Proven guidelines for consumers and designers of multimedia learning*. San Francisco, CA: Pfeiffer.

Cottrell, S., & Donaldson, J. H. (2013). Exploring the opinions of registered nurses working in a clinical transfusion environment on the contribution of e-learning to personal learning and clinical practice: results of a small scale educational research study. *Nurse Education in Practice, 13*(3), 221–227.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: Sage Publications.

Creswell, J. W. (2002). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Upper Saddle River, NJ: Merrill Prentice Hall.

Dillenbourg, P. (2000). Virtual learning environments. In *Proceedings of the EUN Conference 2000—Learning in the new millennium: building new education strategies form schools* (pp. 1–30). Geneva, Switzerland: EUN Conference.

Greefrath, G., & Rieβ, M. (2013). Reality based test tasks with digital tools at lower secondary. In G. A. Stillman & G. Kaiser Blum, W., & J. P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 445–456). New York, NY: Springer.

Guest, G., Bunce, A., & Johnson, L. (2006). How many interviews are enough? An experiment with data saturation and variability. *Field Methods, 18*(1), 59–82.

Haris, D. (2017). *Developing a virtual learning environment on Indonesian version of realística mathematics education*. In: The 2nd Annual International Seminar on Transformative Education and Educational Leadership (AISTEEL-2017). Medan City, Indonesia: Universitas Negeri Medam. Retrieved from http://aisteel2017.unimed.ac.id/. Accessed on 3 Oct 2017.

Holloway, I. (1997). *Basic concepts for qualitative research*. London: Blackwell Science.

Kerr, S. J., Neale, H. R., & Cobb, S. V. (2002). Virtual environments for social skills training: The importance of scaffolding in practice. In *Proceedings of the Fifth International ACM SIGCAPH Conference on Assistive Technologies* (pp. 104–110). New York, NY: ACM Press.

Malheiros, A. P. (2008). *Educação matemática online: A elaboração de projetos de Modelagem* (Online mathematics education: elaborating modelling projects). Unpublished PhD dissertation. Rio Claro, São Paulo, Brazil: UNESP.

Moore, M. G. (1993). Theory of transactional distance. In D. Keegan (Ed.), *Theoretical principles of distance education* (pp. 22–38). New York, NY: Routledge.

Moore, M. G., & Kearsley, G. (2005). *Distance education: A system view*. Belmont, CA: Wadsworth.

O'Reilly, M., & Parker, N. (2012). Unsatisfactory saturation: a critical exploration of the notion of saturated sample sizes in qualitative research. *Qualitative Research, [online], 13*(2), 190–197.

Parra-Zapata, M., Rendón-Mesa, P., Molina-Toro, J., Sánchez-Cardona, J., Ocampo-Arenas, M., & Villa-Ochoa, J. (2017). Participación de profesores en un ambiente de formación online. Ejemplo de un diseño em modelación matemática. Revista Virtual Universidad Católica del Norte, *50* (Febrero-Mayo), 3–20.

Pereira, R. S. G. (2015). *A educação a distância e a formação continuada de professores de matemática: contribuições de um contexto formativo para a base de conhecimento docente* [Distance education and continuing education of mathematics teachers: Contributions of a formative context to the teacher knowledge base]. Doctorate in Mathematics Education. Presidente Prudente: UNESP.

Reese, S. A. (2015). Online learning environments in higher education: Connectivism vs. dissociation. *Education and Information Technologies, 20*(3), 579–588.

Rosa, M. (2000). *From reality to mathematical modelling: A proposal for using ethnomathematical knowledge. College of Education*. Sacramento, CA: California State University, Sacramento (CSUS).

Rosa, M., & Orey, D. C. (2016). Developing mathematical modelling in virtual learning environments by applying critical and reflective dimensions. In K. Wallace (Ed.), *Learning environments: Emerging theories, applications and future directions* (pp. 1–20). New York, NY: Nova Science Publishers.

Rosa, M., & Orey, D. C. (2017). Mathematical modelling in a long-distance teacher education in Brazil: Democratizing mathematics. In G. Stillman & A. Blum, W., & G. Kaiser (Eds.), *Mathematical modelling and applications: Crossing and researching boundaries in mathematics education*. Cham: Springer.

Shepherd, C., Bolliger, D., Dousay, T., & Persichitte, K. (2016). Preparing teachers for online instruction with a graduate certificate program. *TechTrends, 60*(1), 41–47.

Silva, N., Costa, G. J. M., Rogerson, S., & Prior, M. (2007). *Knowledge or content? The philosophical boundaries in e-learning pedagogical theories: Research, reflections and innovations in integrating ICT in education*. London: Routledge.

Silverman, J. (2011). Supporting the development of mathematical knowledge for teaching through online asynchronous collaboration. *Journal of Computers in Mathematics and Science Teaching, 30*(1), 61–78.

Silverman, J. (2012). Exploring the relationship between teachers prominence in online collaboration and the development of mathematical content knowledge for teaching. *Journal of Technology and Teacher Education, 20*(1), 47–69.

Silverstone, S., & Keeler, J. (2013). Engaging students in online marketing classes. *Global Conference on Business and Finance Proceedings, 8*(2), 14–24.

Stake, R. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.

Yin, R. K. (1994). *Case study research: Design and methods*. Newbury Park, CA: Sage Publications.