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A comparative study of the dynamic critical behavior of the four-state Potts like models

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ABSTRACT

We investigate the short-time critical dynamics of the Baxter–Wu (BW) and $n = 3$ Turban (3TU) models to estimate their global persistence exponent θ_g. We conclude that this new dynamical exponent can be useful in detecting differences between the critical behavior of these models which are very difficult to obtain in usual simulations. In addition, we estimate again the dynamical exponents of the four-state Potts (FSP) model in order to compare them with results previously obtained for the BW and 3TU models and to decide between two sets of estimates presented in the current literature. We also revisit the short-time dynamics of the 3TU model in order to check if, as already found for the FSP model, the anomalous dimension of the initial magnetization x_0 could be equal to zero.

1. Introduction

Since the works by Janssen, Schaub and Schmittmann [1], and Huse [2], the critical properties of statistical systems have been a subject of considerable interest in non-equilibrium physics [3–13]. By using renormalization group methods and numerical calculations, respectively, they showed that there is universality and scaling behavior even at the early stage of the time evolution after quenching from high temperatures to the critical one.

The dynamic scaling relation obtained by Janssen et al. [1] for the kth moment of the magnetization, extended to systems of finite size [3], is written as

$$M^{(k)}(t,\tau,L,m_0) = b^{-k\beta/\nu} M^{(k)}(b^{-z}t, b^{1/\nu}\tau, b^{-1}L, b^{x_0}m_0), \tag{1}$$

where t is the time evolution, b is an arbitrary spatial scaling factor, $\tau = (T - T_c)/T_c$ is the reduced temperature and L is the linear size of the lattice. The exponents β and ν are as usual the equilibrium critical exponents associated respectively with the order parameter and the correlation length, z is the dynamical exponent characterizing time correlations in equilibrium, and x_0 represents the anomalous dimension of the initial magnetization m_0, introduced to describe the dependence of the scaling behavior on the initial conditions.

Besides to avoid the well-known problem of the "critical slowing down", characteristic of the equilibrium, and to provide an alternative way to obtain the familiar set of static critical exponents and the dynamic critical exponent z, this kind of investigation reveals a new universal regime and an unsuspected new dynamic critical exponent θ which can be found by

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following the above scaling law for the order parameter at the critical temperature ($\tau = 0$)

$$M(t) \sim m_0 t^{\theta}. \tag{2}$$

This new index, independent of the previously known exponents characterizes the so-called "critical initial slip", the anomalous behavior of the order parameter when a system is quenched to the critical temperature T_c. This exponent is related to x_0 as

$$\theta = \frac{x_0 - \beta/\nu}{z}. \tag{3}$$

Some years later, Majumdar et al. [14] have shown that another dynamic critical exponent can be obtained in the study of systems far from equilibrium. By studying the behavior of the global persistence probability $P(t)$ that the order parameter has not changed its sign up to time t, they have shown that $P(t)$ should behave, at the critical temperature, as

$$P(t) \sim t^{-\theta_g}, \tag{4}$$

where θ_g is the global persistence exponent. They also argued that, if the time evolution of the order parameter would be a Markovian process, then the exponent θ_g should obey the equation [14]

$$\theta_g = \alpha_g = -\theta + \frac{d}{2z} - \frac{\beta}{\nu z}. \tag{5}$$

However, as shown in several works [14–26] the exponent θ_g is an independent critical index closely related to the non-Markovian characteristic of the process.

In this work, we perform short-time Monte Carlo simulations to investigate the scaling behavior of the global persistence probability $P(t)$ for the BW [27,28], 3TU [29,30] and FSP [31,32] models in two dimensions ($d = 2$), that exhibit the same set of leading static critical exponents. We also calculate the exponent x_0 of these models but only after reobtaining more precise estimates for the dynamical indices θ and z related to the FSP and 3TU models. The aim of this paper is to show that it is also possible to detect different behavior between those models by doing short-time Monte Carlo simulations.

The paper is organized as follows. In the next section we present the models. In Section 3 we show the short-time scaling relations and present our results. Finally, in Section 4 we present our conclusions.

2. The models

The q-state Potts model which is a simple extension of the Ising model, has a rich phase diagram [32] with first order phase transitions when $q > 4$ and second order phase transitions when $q \leq 4$. Its Hamiltonian is given by

$$-\beta\mathcal{H} = K \sum_{\langle i,j \rangle} \delta_{\sigma_i \sigma_j}, \tag{6}$$

where $\beta = 1/k_B T$ and k_B is the Boltzmann constant, $\langle i, j\rangle$ represents nearest-neighbor pairs of lattice sites, K is the dimensionless ferromagnetic coupling constant and σ_i is the spin variable which takes the values $\sigma_i = 0, \ldots, q-1$ on the lattice site i. It is well known that the critical coupling of this model is given by Ref. [32]

$$K_c = \log(1 + \sqrt{q}), \tag{7}$$

and its order parameter is defined as

$$M = \frac{1}{L^d(q-1)} \left\langle \sum_i (q\delta_{\sigma_i(t),1} - 1) \right\rangle \tag{8}$$

where L is the linear size of the lattice and d is the dimension of the system. The case $q = 4$ (FSP model) in two dimensions is known to exhibit slow convergence when investigated by finite-size techniques motivated by the presence of a marginal operator (scaling dimension $= d = 2$).

The BW model is defined by the Hamiltonian

$$-\beta\mathcal{H} = K \sum_{\langle i,j,k \rangle} \sigma_i \sigma_j \sigma_k, \tag{9}$$

where $\sigma_i = \pm 1$ is an Ising spin variable located at each site of the triangular lattice and the sum extends over all elementary triangles.

The Hamiltonian of the 3TU model is given by

$$-\beta\mathcal{H} = \sum_{\langle i,j \rangle} \left\{ K_h \sigma_{i,j}\sigma_{i+1,j}\sigma_{i+2,j} + K_v \sigma_{i,j}\sigma_{i,j+1} \right\}, \tag{10}$$

where the sum is over all sites of a square lattice, K_h and K_v are the coupling constant in the horizontal (with three-spin interactions) and vertical (with two-spin interactions) directions, respectively, and $\sigma_{i,j} = \pm 1$ is an Ising spin variable located at each site of the lattice.

Both BW and 3TU (for the isotropic case, $K_h = K_v = K$) models undergo a continuous phase transition at the critical temperature $K_c = 0.5\ln(1+\sqrt{2})$ which is the same critical temperature of the Ising model on a square lattice. The order parameter of these models is defined as

$$M = \frac{1}{L^d}\left\langle \sum_i \sigma_i \right\rangle. \tag{11}$$

The BW and 3TU models present semi-global up-down spin reversal symmetry [33], i.e., their Hamiltonians are invariant under reversal of all the spins belonging to two of three sublattices into which the original lattice can be decomposed.

The ground state of these three models is fourfold degenerated, being that the possible spin configurations of the BW and 3TU models consist of repetitions of the patterns $\{+,+,+\}$, $\{+,-,-\}$, $\{-,+,-\}$ or $\{-,-,+\}$. The main difference between the three models is that the BW model is defined on a triangular lattice whereas in the FSP and 3TU models the spins are located on a square one.

From the degeneracy and symmetry considerations, it was conjectured that these three models would belong to the same universality class, with critical exponents given by Ref. [34]

$$\beta = \frac{1}{12}, \qquad \nu = \alpha = \frac{2}{3}, \quad \text{and} \quad \eta = \frac{1}{4}. \tag{12}$$

However, when these models are deeply studied, differences among sub-dominant exponents appear. These exponents are supposed to be associated to different behavior exhibited by those models when studied by finite-size scaling techniques. This fact was first pointed out by Alcaraz and Xavier [35] in a finite-size scaling study of the FSP and BW models using a conformal invariance approach.

As will be shown in this paper, it is possible to observe remarkable differences between those models by investigating the non-equilibrium evolution of a dynamical quantity introduced by Majumdar et al. [14], the global persistence probability. This result corroborates previous simulations which pointed out different behavior for the BW model when compared to the FSP model [36–38].

3. Results and discussions

In our Monte Carlo simulations, we consider two-dimensional lattices with periodic boundary conditions. The dynamical evolution of the spins is local and updated by the heat-bath algorithm at the critical coupling K_c. In order to check finite-size effects, we consider three different lattice sizes ($L = 120$, 180 and 240) the exponents being obtained from five independent bins of 20 000 samples each one.

3.1. Global persistence exponent θ_g

The global persistence probability $P(t)$ can be defined as

$$P(t) = 1 - \sum_{t'=1}^{t} \rho(t') \tag{13}$$

where $\rho(t')$ is the fraction of the samples that have changed their state for the first time at the instant t'. The dynamical exponent θ_g that governs the behavior of $P(t)$ at criticality is obtained through the power law behavior given by

$$P(t) \sim t^{-\theta_g}. \tag{14}$$

In order to obtain the exponent θ_g, the initial configuration of the system should be carefully prepared with a precise and small value of m_0. After estimating θ_g for a number of m_0 values, its final value is obtained from the limit $m_0 \to 0$. In this work, we used $4 \times 10^{-4} < m_0 \leq 5 \times 10^{-3}$.

As we are considering three models and two different order parameters, it is worth to explain how to obtain m_0 in each case. At the beginning, each site on the lattice of the FSP model is occupied by a spin variable which takes the values $\sigma = 0$, 1, 2 or 3 and, for the 3TU and BW models, the sites are occupied by spin variables which take the values ± 1. For each model, the values of spin variables are chosen with equal probability. Afterward, the magnetization of the models is measured by using Eq. (8) for the FSP model or (11) for the 3TU and BW models. In order to obtain a null value of the initial magnetization, some sites of the lattices are randomly chosen and its signs (or values) are changed. Finally, the desired value for the initial magnetization of each model is obtained by changing the sign (or the value) of δ sites on the lattice. When using Eq. (8), the initial magnetization is given by

$$m_0 = \frac{4\delta}{3L^2} \tag{15}$$

and a value of m_0 is obtained choosing δ sites occupied by $\sigma = 0$, 2 or 3 and substituting them by $\sigma = 1$. For Eq. (11), m_0 is simply given by

$$m_0 = \frac{\delta}{L^2} \tag{16}$$

and to obtain a value of m_0, we choose randomly $\delta/2$ sites occupied by $\sigma = -1$ and substitute them by $\sigma = 1$.

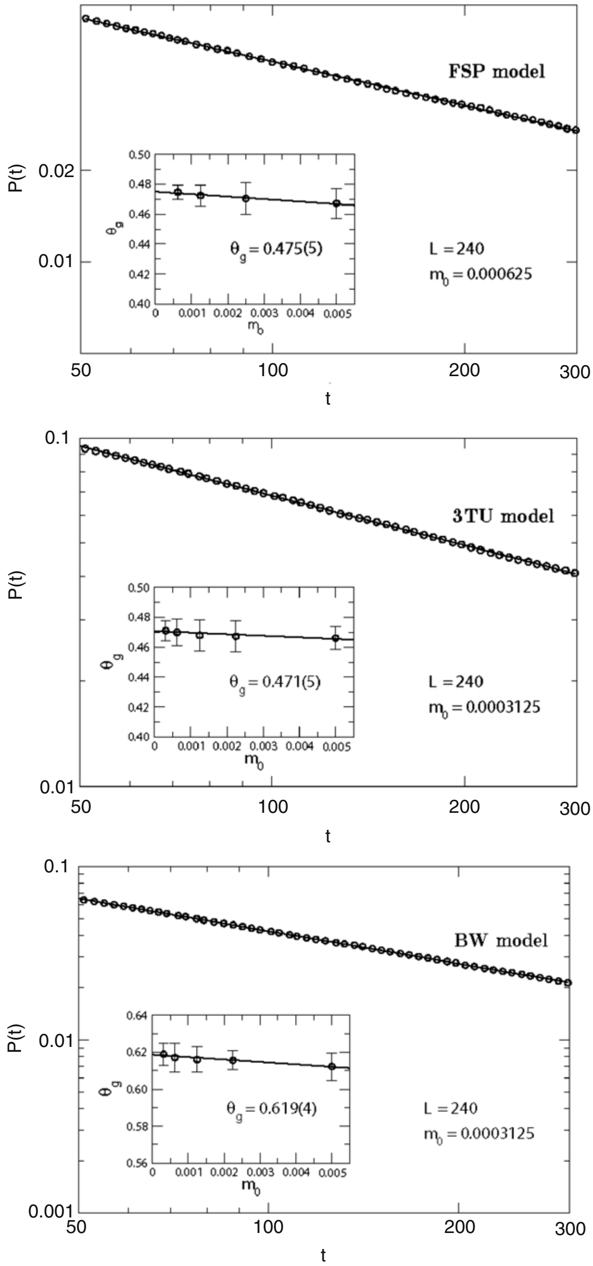


Fig. 1. The time evolution of the global persistence probability $P(t)$ for $L = 240$ for the FSP (on top), 3TU (on middle), and BW (on bottom) models. The error bars calculated over 5 sets of 20 000 samples are smaller than the symbols. The inset in each figure shows the exponent θ_g for different initial magnetizations, as well as its extrapolated value.

In Fig. 1 we show the behavior of the global persistence probability for $L = 240$ and a small value of m_0 for the FSP (on top), 3TU (on middle), and BW (on bottom) models, in double-log scales. The error bars, calculated over five sets of 20 000 samples are smaller than the symbols.

The insets in Fig. 1 display the estimates of θ_g for different values of m_0 and the limiting procedure $m_0 \to 0$ for the models.

In Table 1, we show the extrapolated values of θ_g for $L = 120$, 180 and 240 for the BW, 3TU and FSP models. Finite-size effects are less than statistical errors.

The discrepancy among the results obtained for the persistence exponent for the BW model by one side and for the FSP and 3TU models is noteworthy. Discrepancies between the BW and FSP models in dynamical simulations were previously observed by Arashiro and Drugowich de Felício [37] for the dynamical exponent θ and by Chatelain [38] for the exponent $\lambda/z = d/z - \theta$ and for the asymptotic value of the fluctuation–dissipation ratio X_∞. However, from numerical calculations

Table 1
The global persistence exponent θ_g from the power law behavior for the FSP, 3TU, and BW models.

Models	$L = 120$	$L = 180$	$L = 240$
FSP	0.469(4)	0.472(6)	0.475(5)
3TU	0.469(6)	0.470(5)	0.471(5)
BW	0.620(5)	0.618(5)	0.619(4)

Table 2
The exponent θ for the FSP and 3TU models.

L	FSP model	3TU model
120	−0.046(8)	−0.047(7)
180	−0.047(8)	−0.046(7)
240	−0.046(9)	−0.047(8)

Table 3
The exponent z for the FSP and 3TU models.

L	FSP model	3TU model
120	2.294(7)	2.293(5)
180	2.294(5)	2.290(8)
240	2.296(5)	2.292(4)

made on finite lattices (at the equilibrium) it is well known that BW, FSP and 3TU models show different corrections to finite-size scaling. Whereas estimates for the BW model exhibit good convergence with the system size [35], comparable to that of the two-dimensional Ising model, FSP and 3TU models offer serious barriers to whom wish to find their exponents from finite-size techniques [39,40].

3.2. Dynamic critical exponents θ and z

As conjectured by Janssen et al. [1] on the basis of renormalization group techniques and by Huse [2] through numerical calculations, in the short-time regime, the order parameter obeys a power law as shown in Eq. (2). Formerly, a positive value was always associated to this exponent [5,41–44] and the phenomenon was known as critical initial slip. However, as shown in some papers, there are models in which the exponent θ can have a negative value, for instance, the tricritical Ising model, [45,46], FSP [47], 3TU [48], and BW [37,49] models.

Although the estimates of the exponents θ and z for the BW model are known with good precision, the results for the 3TU model exhibit large error bars. In addition, estimates obtained for θ in previous papers [25,47] show considerable differences between the two techniques employed to study the FSP model. So, in order to obtain more precise estimates for the exponent x_0, we decided to reobtain the exponents θ and z for both FSP and 3TU models by using the time correlation of the magnetization [43]

$$C(t) = \langle M(0)M(t)\rangle \sim t^{\theta} \tag{17}$$

and the function $F_2(t)$ proposed by da Silva et al. [9]

$$F_2(t) = \frac{\langle M^2(t)\rangle_{m_0=0}}{\langle M(t)\rangle^2_{m_0=1}} \sim t^{d/z}. \tag{18}$$

In Eq. (17), the average is taken over a set of random initial configurations. Initially, this approach had shown to be valid only for models which exhibit up-down symmetry [43]. Nevertheless, it has been later found that this approach is more general and can include models with other symmetries [50]. This method has several advantages when compared to other approaches, for instance, the exponent θ can be directly calculated without the need of careful preparation of the initial states nor of the limiting procedure [see Eq. (2)], the only requirement being that $\langle M(0)\rangle = 0$.

In Fig. 2 we show the evolution of the time correlation $C(t)$ in double-log scale for the FSP (on top) and 3TU (on bottom) models, respectively, for $L = 240$.

The slope of these curves is shown in Table 2, as well as the estimates for $L = 120$, 180 and 240.

On the other hand, the dynamical exponent z was obtained by combining results from samples submitted to different initial conditions (see Eq. (18), where $d = 2$ is the dimension of the system). This approach has proved to be very efficient in estimating the exponent z for several models [9,46,37,44]. The time evolution of F_2 is shown on log scales in Fig. 3 for $L = 240$ for the FSP (on top) and 3TU (on bottom) models.

Taking into account the values of the ratio d/z, estimated from the slope of these curves, the exponent z can be easily found. Our estimates for this exponent for the FSP and 3TU models are shown in Table 3 for $L = 120$, 180, and 240.

As shown in Table 3, the estimates for the dynamical exponent z of the FSP and 3TU models are in complete agreement with our results for the BW model [37] ($z = 2.294(6)$). However, the values we found for the dynamical exponent θ of the

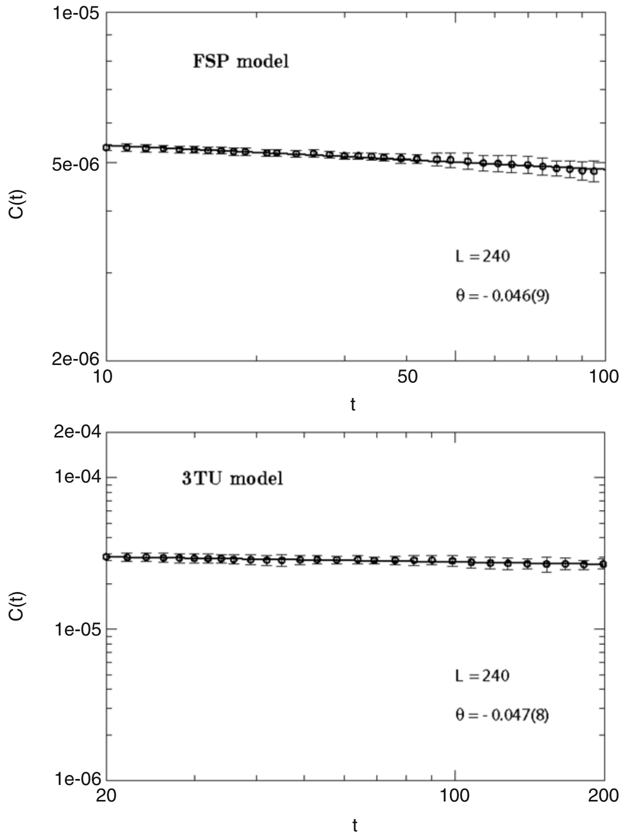


Fig. 2. The time correlation of the order parameter on log scales for the FSP (on top) and 3TU (on bottom) models. Error bars were calculated over 5 sets of 20 000 samples.

Table 4
The exponent α_g for the FSP, 3TU, BW models.

Models	α_g
FSP	0.427(10)
3TU	0.429(9)
BW [37]	0.567(3)

FSP and 3TU models (Table 2) are completely different from the previously estimated exponent for the BW model [37]

$$\theta = -0.186(2). \tag{19}$$

3.3. The exponent α_g and the anomalous dimension x_0

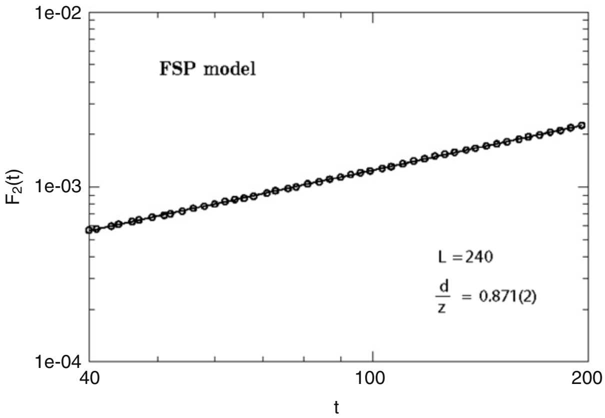
Using the results of Tables 2 and 3 for $L = 240$ (FSP and 3TU models), the results of Eq. (19) and the values of β and ν of Eq. (12) we estimate the exponent α_g through Eq. (5) for the studied models (see Table 4). The difference between our estimate for θ_g (See Table 1) and the value obtained from Eq. (5) shows the non-Markovian aspect for the BW, 3TU and FSP models. Thus, the global persistence exponent in these cases is also independent of other critical exponents.

We remark that using the estimates of Hadjiagapiou et al. [49] for the dynamical exponents of the BW model $z = 1.994(24)$ and $\theta = -0.185(2)$ we obtain $\alpha_g = 0.624(3)$ approximately equal to θ_g (from Table 1) which means that the relaxation would be Markovian. As we know, the models studied until now [14–26] exhibit different values for those exponents and this would be the first case where Eq. (5) would be valid.

Finally, we calculate the value of the anomalous dimension x_0 of the order parameter for the FSP, 3TU, and BW models. This exponent, which is introduced to describe the dependence on the scaling behavior of the initial conditions, is related to the exponents θ, z, and β/ν by Eq. (3). So,

$$x_0 = \theta z + \beta/\nu. \tag{20}$$

In Table 5, we show the estimates of x_0 by using the values of Tables 2 and 3 for $L = 240$ (FSP and 3TU models), the estimates for θ [37,49] and the conjectured values of β and ν given by Eq. (12).



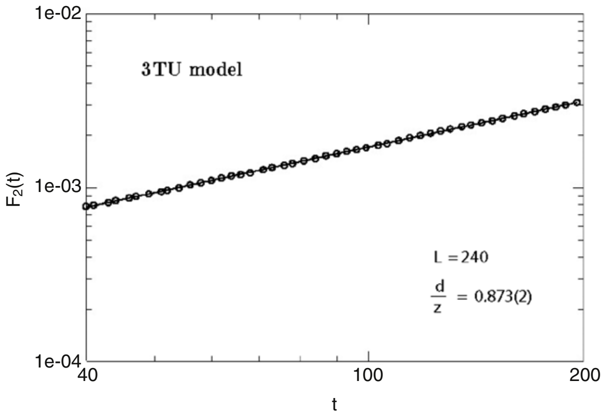


Fig. 3. The time evolution of $F_2(t)$ for $L = 240$ for the FSP (on top) and 3TU (on bottom) models. Each point represents an average over 5 sets of 20 000 samples and the error bars are obtained of them.

Table 5
The exponent x_0 for the FSP, 3TU, BW models.

Models	x_0
FSP	0.019(21)
3TU	0.017(18)
BW [37]	−0.302(6)
BW [49]	−0.244(8)

As we can see in Table 5, our results indicate that far from equilibrium the critical behavior of the FSP and 3TU models is very similar but different from the BW one. In addition, our estimates do not exclude a null value for the anomalous dimension of the magnetization (x_0) in those cases (FSP and 3TU models) which in static critical phenomena theory is known to be associated to marginal operators [51] and in finite-size scaling calculations to logarithmic corrections [52,53].

4. Conclusions

We estimated the dynamical exponent θ_g for the BW, 3TU, and FSP models using the time evolution of the global persistence probability that the magnetization has not changed its signal up to time t. The value of θ_g found for the BW is completely different from that found for the 3TU and FSP models. On the other hand our results for the FSP and 3TU models are in good agreement with each other. As previously found for the exponent θ of the initial magnetization, the persistence exponent of Majumdar et al. [14] is also able to detect differences between the BW and the 3TU and FSP models. We stress that those differences are very difficult to obtain in static critical phenomena study. We also reobtained the dynamical exponent θ for the 3TU and FSP models. In the case of the 3TU model we found much more precise values than previously found by Simões and Drugowich de Felicio [48] in consequence of better statistics whereas present results for the FSP model only confirm estimates recently found by Fernandes et al. [25] using a different order parameter introduced by Vanderzande [54]. Finally, with new and more precise estimates for the exponent θ we could recalculate the anomalous dimension x_0 of the initial magnetization for the three models and the conclusion is that the value zero (related to a marginal operator in the language of the renormalization group) cannot be discarded for FSP and 3TU models but is completely unlikely for the BW model.

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